

# Transients on Lossless Exponential Transmission Lines Using Allen's Method

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**Abstract**—The exact analytical expressions of the time-domain step response matrix for the lossless exponential transmission line are developed, therefore extending the range of problems where Allen's method can be applied for the transient analysis of networks consisting of interconnections of linear distributed elements, lumped linear and/or nonlinear elements, and arbitrary sources. An indication as to the correctness of the expressions is obtained by comparing them to published results, which also helps to gain a better physical insight into the step response matrix. Moreover, the response to a step input and also the transient response to a sudden sinusoidal excitation are presented.

## I. INTRODUCTION

DESPITE its interest, the transient behavior of nonuniform two-conductor transmission lines has been addressed only by a limited number of papers [1]–[9]. These authors have studied a single line driven by a voltage or a current source and terminated, in most cases, by a linear load resistance. The input signal is an ideal step function  $u(t)$ , with the exception of [2] and [3], where use is made of a trapezoidal pulse and a step with a finite rise time, respectively. Two basic approaches are considered: on one hand, the method of characteristics is applied to a nonuniform line approximated by sections of uniform lines [1]–[3]; on the other hand, the Laplace transform is used to obtain closed-form time-domain solutions for specific types of tapered lines and special end conditions [4]–[6]. These latter expressions are extremely complicated and require numerical techniques for any quantitative evaluation [e.g., 4, Table 1]; furthermore, they are only valid for a limited time span (typically,  $\tau \leq t < 3\tau$ , where  $\tau$  is the one-way transit time of the line). More recently, just as in [1]–[3], Hsue and Hechtman [7] approximated a nonuniform line with a cascade of uniform line segments and obtained its step response by summing the multiple reflections instead of using the method of characteristics. This approach, just as in [1]–[3], requires a large amount of line segments for an adequate simulation of fast transients, and is therefore unattractive in terms of computer memory requirements and execution time. Recently, Hsue [8] studied the lossless exponential line. His work was exclusively concerned with the derivation of the time-domain scattering parameters of that particular line using the inverse Laplace transform. The author did not mention which technique could be used to compute the corresponding

transient voltage and current waveforms from the scattering parameters and the boundary conditions of the line. Finally Chang [9], combining the method of characteristics and the waveform relaxation technique, studied among other things the step response of a parabolic nonuniform line with skin-effect losses, for linear and nonlinear loads.

In this paper, transients on lossless exponential lines are studied following Silverberg and Wing's approach [10] (revised by Allen [11]) which was originally applied to lossless and lossy but distortionless uniform lines. This numerical method can be used for the transient analysis of networks consisting of interconnections of linear distributed elements, lumped linear and/or nonlinear elements, and arbitrary independent or dependent sources. The overall network is solved in the time domain using convolution techniques. According to [11], each linear subnetwork is characterized *in the time domain* by a step response matrix  $\vec{a}(t)$ .

The inverse Laplace transform is used to obtain the exact analytical expressions of the  $\vec{a}(t)$  matrix for the lossless exponential line, thus widening the range of application of Allen's method. Moreover, complete transient responses using these expressions are presented for a step signal with a finite rise time and, also, for a sinusoidal input. The effects of multiple reflections from both ends of the line are clearly visible, even though the receiving end is terminated in a “matched load.”

## II. TIME-DOMAIN STEP RESPONSE MATRIX OF A LOSSLESS EXPONENTIAL LINE

Ghausi and Kelly [12] obtained the  $Y$  parameters (short-circuit admittances) in closed form for a class of nonuniform distributed networks in which the per-unit-length series impedance  $Z(s, z)$  and the per-unit-length shunt admittance  $Y(s, z)$  can be separated into functions of the Laplace transform variable  $s$  and the distance  $z$  along the line; moreover, the product  $Z(s, z)Y(s, z)$  is independent of  $z$ . In other words, these authors restricted their study to networks such that

$$Z(s, z) = \frac{Z(s)}{p(z)} \quad \text{and} \quad Y(s, z) = Y(s)p(z) \quad (1)$$

where  $Z(s) = sL_0$  and  $Y(s) = sC_0$  for a lossless line.  $P(z)$  describes the variations of the distributed inductance and capacitance along the line. Thus, for the exponential line,

$$p(z) = e^{\pm 2\delta z} \quad \delta \geq 0 \quad (2)$$

where  $\delta$  is the taper rate. For negative values of the argument in (2), the per-unit-length inductance *increases* exponentially

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with the distance from its initial value  $L_0$  at  $z = 0$  and, similarly, the per-unit-length capacitance *decreases* exponentially with  $z$  from its initial value  $C_0$  at  $z = 0$ . Finally, the "characteristic impedance" of the exponential line can be written, using (1) and (2), as

$$Z_0(s, z) = \sqrt{\frac{Z(s, z)}{Y(s, z)}} = \sqrt{\frac{L_0}{C_0}} e^{2\delta z} = Z_0(0) e^{2\delta z} \quad (3)$$

where  $Z_0(0)$  is the impedance at  $z = 0$ .

Substitution of the above expressions into eq. (4-16) of [12] yields the short-circuit admittance parameters of the exponential line in the Laplace transform domain [10], [11]

$$Y_{11}(s) = \frac{1}{s\ell L_0} \left[ \ell \sqrt{s^2 L_0 C_0 + \delta^2} \right. \\ \left. \cdot \coth(\ell \sqrt{s^2 L_0 C_0 + \delta^2}) - \delta \ell \right] \quad (4a)$$

$$Y_{12}(s) = Y_{21}(s) = \frac{-e^{-\delta\ell}}{s\ell L_0} \ell \sqrt{s^2 L_0 C_0 + \delta^2} \\ \cdot \operatorname{csch}(\ell \sqrt{s^2 L_0 C_0 + \delta^2}) \quad (4b)$$

$$Y_{22}(s) = \frac{e^{-2\delta\ell}}{s\ell L_0} \left[ \ell \sqrt{s^2 L_0 C_0 + \delta^2} \right. \\ \left. \cdot \coth(\ell \sqrt{s^2 L_0 C_0 + \delta^2}) + \delta \ell \right] \quad (4c)$$

where  $\ell$  is the length of the line in meters.

Thus, for a nonuniform line, the reciprocity property  $Y_{12}(s) = Y_{21}(s)$  is valid, as it is in the uniform case; whereas the symmetry property  $Y_{11}(s) = Y_{22}(s)$  is lost [13]. As a consequence, three parameters must be evaluated instead of two.

The goal here is to obtain the step response matrix  $\vec{a}(t)$  in the time domain, which is related to the short-circuit admittance matrix in the following manner [11]:

$$\vec{a}(t) = \mathcal{L}^{-1}\{\vec{A}(s)\} = \mathcal{L}^{-1}\left\{(1/s)\vec{Y}(s)\right\}. \quad (5)$$

It is thus necessary to evaluate the inverse Laplace transform of

$$A_{11}(s) = \theta \left[ \frac{\tau \sqrt{s^2 + \alpha^2}}{s^2} \coth(\tau \sqrt{s^2 + \alpha^2}) - \frac{\varphi}{s^2} \right] \quad (6a)$$

$$A_{12}(s) = A_{21}(s) \\ = -\theta e^{-\delta\ell} \frac{\tau \sqrt{s^2 + \alpha^2}}{s^2} \operatorname{csch}(\tau \sqrt{s^2 + \alpha^2}) \quad (6b)$$

$$A_{22}(s) = \theta e^{-2\delta\ell} \left[ \frac{\tau \sqrt{s^2 + \alpha^2}}{s^2} \coth(\tau \sqrt{s^2 + \alpha^2}) + \frac{\varphi}{s^2} \right] \quad (6c)$$

where

$$\alpha = \frac{\delta}{\sqrt{L_0 C_0}} \quad (\text{rad/s}) \quad (7)$$

$$\theta = \frac{1}{\ell L_0} \quad (\text{H}^{-1}) \quad (8)$$

$$\tau = \ell \sqrt{L_0 C_0} \quad (\text{s}) \quad (9)$$

$$\varphi = \delta \ell \quad (\text{dimensionless}). \quad (10)$$

It might be of interest to note that (7) represents the cutoff frequency of the line in the sinusoidal steady state.

With the exception of  $\varphi/s^2$ , which is the transform of the ramp function  $\varphi t u(t)$ , the inverse transform of the elements  $A_{ij}(s)$  in (6) cannot be found directly even in two of the most complete tables of Laplace transforms [14], [15]. Accordingly, it seems appropriate to show how the inverse Laplace transform of these expressions were obtained.

The evaluation of  $a_{11}(t) = \mathcal{L}^{-1}\{A_{11}(s)\}$  is considered first. Since [16]

$$\coth x = 2 \sum_{n=0}^{\infty} \varepsilon_n e^{-2nx} \quad \operatorname{Re}(x) > 0 \quad (11)$$

where

$$\varepsilon_n = \begin{cases} 1/2, & n = 0 \\ 1, & n > 0 \end{cases} \quad (12)$$

(6a) can be expanded into a series

$$A_{11}(s) = \theta \left[ \frac{2\tau \sqrt{s^2 + \alpha^2}}{s^2} \sum_{n=0}^{\infty} \varepsilon_n e^{-2n\tau \sqrt{s^2 + \alpha^2}} - \frac{\varphi}{s^2} \right]. \quad (13)$$

The  $n$ th term of this series can be rewritten as

$$\mathcal{L}^{-1} \left\{ \frac{\sqrt{s^2 + \alpha^2}}{s^2} e^{-b\sqrt{s^2 + \alpha^2}} \right\} = \mathcal{L}^{-1} \left\{ \frac{e^{-b\sqrt{s^2 + \alpha^2}}}{\sqrt{s^2 + \alpha^2}} \right\} \\ + \alpha^2 \mathcal{L}^{-1} \left\{ \frac{e^{-b\sqrt{s^2 + \alpha^2}}}{s^2 \sqrt{s^2 + \alpha^2}} \right\}. \quad (14)$$

The first term on the right side of (14) is readily recognized as [14, p. 250, #3.2-46]

$$\mathcal{L}^{-1} \left\{ \frac{e^{-b\sqrt{s^2 + \alpha^2}}}{\sqrt{s^2 + \alpha^2}} \right\} = J_0(\alpha \sqrt{t^2 - b^2}) u(t - b) \quad b > 0 \quad (15)$$

where  $J_0(y)$  is the Bessel function of the first kind of index zero [15]. The following property [15, p. 1020]:

$$\mathcal{L}^{-1} \left\{ \frac{F(s)}{s^2} \right\} = \int_0^t \int_0^x f(\xi) d\xi dx = \int_0^t (t - x) f(x) dx \quad (16)$$

can be applied to the last term of (14) which, combined with (15), yields

$$\mathcal{L}^{-1} \left\{ \frac{\sqrt{s^2 + \alpha^2}}{s^2} e^{-b\sqrt{s^2 + \alpha^2}} \right\} = \left[ J_0(\alpha \sqrt{t^2 - b^2}) \right. \\ \left. + \alpha^2 \int_b^t (t - x) J_0(\alpha \sqrt{x^2 - b^2}) dx \right] u(t - b). \quad (17)$$

This expression can be rewritten as

$$\begin{aligned} \mathcal{L}^{-1} \left\{ \frac{\sqrt{s^2 + \alpha^2}}{s^2} e^{-b\sqrt{s^2 + \alpha^2}} \right\} &= \left[ J_0(\alpha\sqrt{t^2 - b^2}) \right. \\ &+ \alpha^2 t \int_b^t J_0(\alpha\sqrt{x^2 - b^2}) dx \\ &\left. - \alpha\sqrt{t^2 - b^2} J_1(\alpha\sqrt{t^2 - b^2}) \right] u(t-b). \quad (18) \end{aligned}$$

The integrands in (17) and (18) are zero for  $x < b$ , because of the presence of the unit step function  $u(t-b)$  in (15). Taking the inverse transform of (13) term by term, using (18) with  $b = 2n\tau$ , and reworking constants (8)–(10), yields

$$\begin{aligned} a_{11}(t) &= Y_0(0) \left\{ 2 \sum_{n=0}^{\infty} \varepsilon_n \left[ J_0(\alpha\sqrt{t^2 - (2n\tau)^2}) \right. \right. \\ &+ \alpha^2 t \int_{2n\tau}^t J_0(\alpha\sqrt{x^2 - (2n\tau)^2}) dx - \alpha\sqrt{t^2 - (2n\tau)^2} J_1 \\ &\left. \cdot \left( \alpha\sqrt{t^2 - (2n\tau)^2} \right) \right] u(t-2n\tau) - \alpha t u(t) \left. \right\} \quad (19) \end{aligned}$$

where  $Y_0(0)$  is the “characteristic admittance” at the sending end of the line.

The evaluation of  $a_{22}(t)$  follows a line similar to that used for  $a_{11}(t)$ , which gives

$$\begin{aligned} a_{22}(t) &= Y_0(0) e^{-2\delta t} \left\{ 2 \sum_{n=0}^{\infty} \varepsilon_n \left[ J_0(\alpha\sqrt{t^2 - (2n\tau)^2}) \right. \right. \\ &+ \alpha^2 t \int_{2n\tau}^t J_0(\alpha\sqrt{x^2 - (2n\tau)^2}) dx - \alpha\sqrt{t^2 - (2n\tau)^2} J_1 \\ &\left. \cdot \left( \alpha\sqrt{t^2 - (2n\tau)^2} \right) \right] u(t-2n\tau) + \alpha t u(t) \left. \right\}. \quad (20) \end{aligned}$$

This section is closed with the evaluation of  $a_{12}(t)$ . Since [16]

$$\operatorname{csch} x = 2 \sum_{n=1}^{\infty} e^{-(2n-1)x} \quad \operatorname{Re}(x) > 0 \quad (21)$$

(6b) can also be expressed as a series expansion

$$\begin{aligned} A_{12}(s) &= A_{21}(s) \\ &= -2\theta e^{-\delta t} \frac{\tau\sqrt{s^2 + \alpha^2}}{s^2} \sum_{n=1}^{\infty} e^{-(2n-1)\tau\sqrt{s^2 + \alpha^2}}. \quad (22) \end{aligned}$$

Taking the inverse transform of (22) term by term, using (18) with  $b = (2n-1)\tau$ , readily gives

$$\begin{aligned} a_{12}(t) &= -2Y_0(0) e^{-\delta t} \sum_{n=1}^{\infty} \left[ J_0(\alpha\sqrt{t^2 - (2n-1)^2\tau^2}) \right. \\ &+ \alpha^2 t \int_{(2n-1)\tau}^t J_0(\alpha\sqrt{x^2 - (2n-1)^2\tau^2}) dx \\ &\left. - \alpha\sqrt{t^2 - (2n-1)^2\tau^2} J_1(\alpha\sqrt{t^2 - (2n-1)^2\tau^2}) \right] \\ &\cdot u(t - (2n-1)\tau) = a_{21}(t). \quad (23) \end{aligned}$$

This terminates the evaluation of the four elements of the step response matrix for a lossless exponential line.

### III. AGREEMENT WITH COMPARABLE RESULTS

An indication as to the validity of the expressions presented above can be obtained by relating them to comparable results.

*First Case:* We shall consider the lossless uniform line. Clearly,  $\delta = 0$  in this case [see (2)] and, from (7) and (10), it is seen that  $a = \varphi = 0$ . Consequently, the step response matrix of the lossless uniform line, given by Allen [11, Fig. 7b], can be obtained as a special case of (19), (20), and (23) by a simple substitution of these particular values of  $a$  and  $\delta$ .

*Second Case:* Schatz and Williams [4, Table 1] derived analytical expressions, valid for a limited time span, for the ends' current of a lossless exponential line driven by an ideal step generator and terminated by a short circuit. It will be shown that these expressions correspond to particular cases of the series presented by the authors. A few relations must be established first.

A linear two-port characterized by the step response matrix defined by (5) has the following relationship between the currents and the voltages in the Laplace transform domain:

$$I_1(s) = A_{11}(s) \cdot sV_1(s) + A_{12}(s) \cdot sV_2(s) \quad (24a)$$

$$-I_2(s) = A_{21}(s) \cdot sV_1(s) + A_{22}(s) \cdot sV_2(s) \quad (24b)$$

with both currents entering the two-port. For the particular situation where the output port is short-circuited and the input port is driven by an ideal unit step  $u(t)$  generator (with zero internal impedance), (24a) and (24b) reduce to

$$I_1(s) = A_{11}(s) \cdot s \cdot (1/s) = A_{11}(s) \quad (25a)$$

$$-I_2(s) = A_{21}(s) \cdot s \cdot (1/s) = A_{12}(s). \quad (25b)$$

Thus, for a lossless exponential line subjected to these boundary conditions,  $i_1(t)$  and  $-i_2(t)$  will be given by (19) and (23), respectively, in the range  $0^+ \leq t < \infty$ .

Now let us consider the expression given by Schatz and Williams [4, Table 1, 2nd row] for the sending end current (valid for  $0^+ \leq t < 2\tau$ ). With the help of [15, p. 480]

$$\int_0^y J_0(x) dx = y J_0(y) + \frac{\pi y}{2} \{ J_1(y) H_0(y) - J_0(y) H_1(y) \} \quad (26)$$

where  $H_\nu(y)$  is the Struve function of order  $\nu$  [15], Schatz and Williams' expression can be rewritten as

$$\begin{aligned} i(0, t) &= Y_0(0) \left\{ J_0(\alpha t) + \alpha t \left[ \int_0^{\alpha t} J_0(x) dx - J_1(\alpha t) - 1 \right] \right\} \\ &\cdot u(t) \quad (27) \end{aligned}$$

which is identical to the first term,  $n = 0$ , of the series (19) defining  $a_{11}(t)$ . With respect to the receiving end current, the expression presented by Schatz and Williams [4, Table 1], valid in the range  $\tau \leq t < 3\tau$ , is

$$\begin{aligned} i(\ell, t) &= 2Y_0(0) e^{-\delta t} \left[ J_0(\alpha\sqrt{t^2 - \tau^2}) + \alpha^2 t \int_{\tau}^t J_0 \right. \\ &\cdot \left. (\alpha\sqrt{x^2 - \tau^2}) dx - \alpha\sqrt{t^2 - \tau^2} J_1(\alpha\sqrt{t^2 - \tau^2}) \right] u(t - \tau). \quad (28) \end{aligned}$$

It is easily verified that this equation is identical (except for a minus sign) to the first term,  $n = 1$ , of the series (23) defining  $a_{12}(t)$ . These results confirm that the two expressions given by Schatz and Williams, which were considered here, are particular cases of those presented in this paper.

#### IV. CONSIDERATIONS ON THE NUMERICAL EVALUATION

The Bessel functions were computed using the routines found in [17]. The indefinite integrals in (19), (20), (23) require numerical techniques for their evaluation and the composite trapezoidal rule (e.g., algorithm *QTrap* in [17]) was found to be well suited for that task.

#### V. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the possible uses of the matrix obtained in Section II, the step response (Fig. 1) and also the transient response to a sinusoidal input (Fig. 2) are presented. The

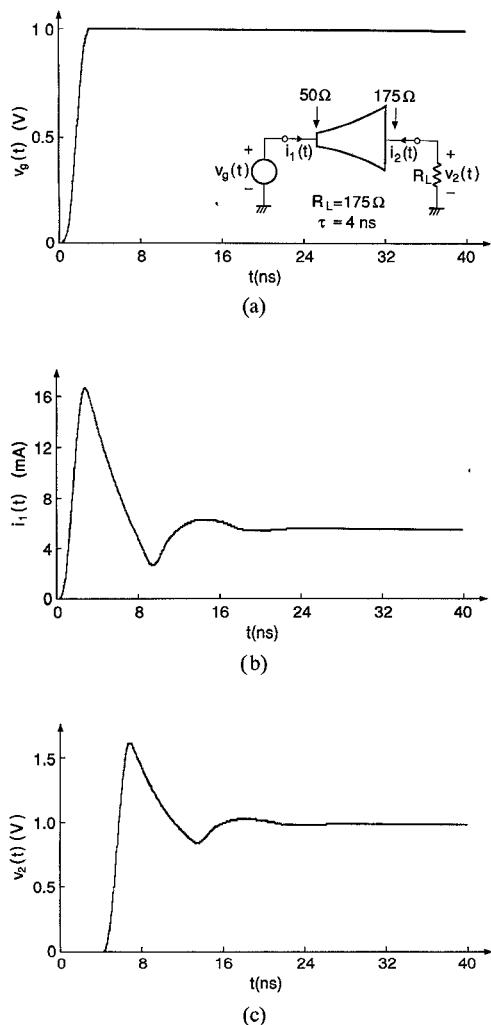


Fig. 1. Step response waveforms for the exponential line network. (a) Generator voltage:  $v_g(t) = I_x(4, 4)$  with  $x = t/(80\Delta)$ , for  $0^+ \leq t \leq 80\Delta$ , and  $v_g(t) = 1$  for  $t > 80\Delta$ ; (b) sending end current; (c) receiving end voltage.

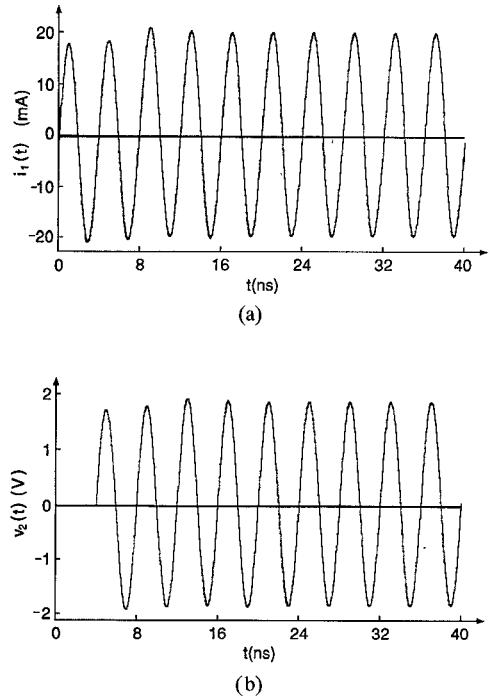


Fig. 2. Sinusoidal transient waveforms for the exponential line network. (a) Sending end current; (b) receiving end voltage.

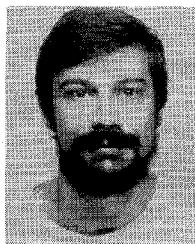
lossless exponential line is driven by a voltage source and is terminated by a resistive load whose value is equal to the "characteristic impedance" at  $z = \ell$  [see (3)]. The line is 0.4 m long and is characterized by the parameters  $L_0 = 0.5 \mu\text{H/m}$  and  $C_0 = 200 \text{ pF/m}$ . The "characteristic impedance"  $Z_0(s, z)$  is, respectively,  $50 \Omega$  at the input port and  $175 \Omega$  at the output end. Allen's method has been applied to compute the transient response. In this case, the network is composed of a single two-port, the exponential line, which is subjected to the boundary conditions  $v_1(t) = v_g(t)$  at  $z = 0$ , and  $v_2(t) = R_L i_2(t)$  at  $z = \ell$ . The unit step is approximated by the incomplete Beta function [17]:  $v_g(t) = I_x(4, 4)$ . Its rise time, from 10 to 90% of its final value, is very nearly 1.4 ns [see Fig. 1(a)]. The harmonic signal is given by  $v_g(t) = \sin \omega_0 t$ , where  $f_0 = 1/\tau = 250 \text{ MHz}$ ; for this frequency, the line is exactly one wavelength long in the steady state. Figs. 1 and 2 were obtained for  $0^+ \leq t \leq 10\tau$ , with a time resolution  $\Delta = \tau/100$  [11]. Unlike the uniform line, the exponential line is not matched for every type of signal by merely forcing the equality between  $R_L$  and the "characteristic impedance" at the receiving end. Indeed, the effects of multiple reflections can be seen in Fig. 1, for a step excitation. With respect to the response to a sudden sinusoidal excitation (Fig. 2), a short transient is also observed, indicating multiple reflections. In the sinusoidal steady state, the ratio of the output voltage to the input voltage is 1.88, which agrees well with the theoretical value of 1.87 for the step-up ratio [4]. The slight discrepancy can be attributed to the fact that a perfect match would require a series  $RC$  load. But since the line is operated far above its cutoff frequency of about 25 MHz [see (7)], it is very nearly matched with only a resistive load impedance equal to  $Z_0(\ell)$  [18].

## VI. CONCLUSION

The exact analytical expressions of the step response matrix for the lossless exponential transmission line have been developed in the time domain, therefore extending the range of problems where Allen's method can be applied. This approach can be used to compute the transient response of networks containing lossless exponential lines subjected to arbitrary sources and quite general boundary conditions, for any time span of interest. An indication as to the correctness of these expressions has been obtained by relating them to published results; moreover, the response to a step input and, also, to a sudden sinusoidal excitation have been presented.

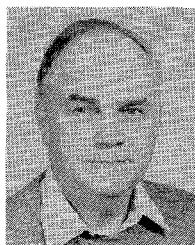
## REFERENCES

- [1] V. Dvorak, "Transient analysis of nonuniform transmission lines," *Proc. IEEE*, vol. 58, pp. 844-845, May 1970.
- [2] —, "Computer simulation of signal propagation through a nonuniform transmission line," *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 580-583, Sept. 1973.
- [3] J. L. Hill and D. Mathews, "Transient analysis of systems with exponential transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 777-783, Sept. 1977.
- [4] E. R. Schatz and E. M. Williams, "Pulse transients in exponential transmission lines," *Proc. IRE*, vol. 38, pp. 1208-1212, Oct. 1950.
- [5] H. Curtins, J. J. Max, and A. V. Shah, "Step response of lossless parabolic transmission line," *Electron. Lett.*, vol. 19, pp. 755-756, Sept. 15, 1983.
- [6] H. Curtins and A. V. Shah, "Step response of lossless nonuniform transmission lines with power-law characteristic impedance function," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1210-1212, Nov. 1985.
- [7] C.-W. Hsue and C. D. Hechtman, "Transient analysis of nonuniform, high-pass transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-38, pp. 1023-1030, Aug. 1990.
- [8] C.-W. Hsue, "Time-domain scattering parameters of an exponential transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 1891-1895, Nov. 1991.
- [9] F. Y. Chang, "Waveform relaxation analysis of nonuniform lossy transmission lines characterized with frequency-dependent parameters," *IEEE Trans. Circuits Syst.*, vol. CAS-38, pp. 1484-1500, Dec. 1991.
- [10] M. Silverberg and O. Wing, "Time domain computer solutions for networks containing lumped nonlinear elements," *IEEE Trans. Circuit Theory*, vol. CT-15, pp. 292-294, Sept. 1968.
- [11] J. L. Allen, "Time-domain analysis of lumped-distributed networks," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 890-896, Nov. 1979.
- [12] M. S. Ghausi and J. J. Kelly, *Introduction to Distributed-Parameter Networks with Applications to Integrated Circuits*. New York: Holt, Rinehart, Winston, 1968, pp. 115-121.
- [13] L. N. Dworsky, *Modern Transmission Line Theory and Applications*. New York: Wiley, 1979, p. 37.
- [14] G. E. Roberts and H. Kaufman, *Table of Laplace Transforms*. Philadelphia: W. B. Saunders, 1966.
- [15] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, 9th ed. New York: Dover, 1972, ch. 29, 9, 11, and 12.
- [16] W. H. Beyer, *CRC Standard Mathematical Tables*, 25th ed. West Palm Beach, FL: CRC Press, 1978, pp. 257-258.
- [17] W. H. Press *et al.*, *Numerical Recipes: The Art of Scientific Computing*. Cambridge: Cambridge University Press, 1988, ch. 4 and 6.
- [18] W. C. Johnson, *Transmission Lines and Networks*. New York: McGraw-Hill, 1950, pp. 205-208.



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